Gamma photons from parametric resonance in neutron stars

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Shock waves in cold nuclear matter, e.g., those induced by a collision of two neutron stars, can generate a large number of gamma photons via parametric resonance. We study the resonant production of gamma rays inside a shocked neutron star and discuss the possible astrophysical consequences of this phenomenon. [S0556-2821(99)02912-4]

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I. INTRODUCTION

Neutron star collisions rank among the most energetic events expected to take place in the Universe, which makes them a natural candidate for the source of observed gamma-ray bursts (GRBs) [1]. The discovery of the afterglow [2] associated with some of the GRBs and the isotropy of the GRBs both support the hypothesis that the GRBs originate at cosmological distances with a redshift of order 1. The total energy $\sim 10^{53}$ erg released when the two neutron stars merge is sufficient to explain the GRBs, although it is not clear what fraction of that energy is converted to gamma rays. Another puzzling feature of the GRBs is their non-thermal spectrum.

Understanding the physics of the explosion that follows the collision of two neutron stars is of great importance and is the subject of intense studies [3]. The approach and the early stages of the interaction of the two neutron stars are accompanied by powerful acoustic shock waves that propagate through nuclear matter and eventually dissipate their energy into heat. During this period of time, repetitive superconducting phase transitions can take place in part of the star's volume due to the sharp dependence of the proton energy gap on density. The relaxation of the proton condensate to the potential minimum can be accompanied by a non-thermal resonant production of gamma rays [4] in the MeV energy range. If the parametric resonance is efficient, the power transferred to the coherent gamma quanta may exceed 10^{52} erg/s. Eventually, the nuclear matter is heated above the critical temperature ~ 0.5 MeV and the production of the gamma rays comes to a halt.

The photons produced inside a neutron star cannot decay through pair production $\gamma \to e^+ e^-$ because the degenerate electrons have a chemical potential in excess of 100 MeV, far greater than the photon energy. The decay into electron-positron pairs is, therefore, prohibited by the Pauli exclusion principle. The photons undergo a Compton scattering off the electrons near the Fermi surface which is sufficiently strong to keep the photons from escaping. While such scattering (or Comptonization) may change the spectrum of the gamma component, the number of photons remains the same. Since extreme electron degeneracy is maintained even at the highest temperatures achieved in the fireball [3], the gamma-ray component is present in nuclear matter at the time the latter is dispersed by the explosion.

The latter can have several consequences. In particular, γ quanta, abundantly present in nuclear matter at the onset of the explosion, can be released when the fireball erupts. A detailed investigation of this signal lies outside the scope of this paper. However, it is plausible that the gamma rays emitted at that point would have a non-thermal spectrum. At later times, when the fireball reaches a high temperature, other sources of gamma emission become dominant. We predict, therefore, a qualitative difference in the spectrum of gamma rays emitted during the first milliseconds of the collision.

In this paper we will not attempt to understand the emission of gamma rays from the fireball. Instead, we will concentrate on the phenomenon of the resonant production of photons which transfers a fraction of the gravitational energy into the gamma quanta inside a neutron star. This process is of fundamental interest on its own and may have important consequences.

II. BASIC IDEA

Nuclear matter in the interior of a neutron star may be superconducting. The existence of the superfluid proton condensate depends on several theoretical assumptions, some of which may be hard to justify. The proper use of many-body techniques and the choice of macroscopic degrees of freedom are by no means obvious. However, assuming that the theoretical framework of Refs. [5–7] is valid, one can elucidate some generic features of proton superfluidity. The most important one for us is that the energy gap depends sharply on density, as shown in Fig. 1. An acoustic shock wave passing through a star can produce significant changes in the density and cause repetitive superconducting phase transitions occurring with periods of the order of the acoustical time scale $\tau_a \sim 10^{-7}$–10^{-3} s [8]. The relaxation time of the proton condensate is of order MeV^{-1}, or 10^{-20} s$\ll \tau_a$, and therefore the system quickly settles into the potential minimum after the passage of each shock wave.

In the presence of a magnetic field the superconducting phase transition is first order [5] and occurs in two stages. First, a bubble of the superconducting phase nucleates and

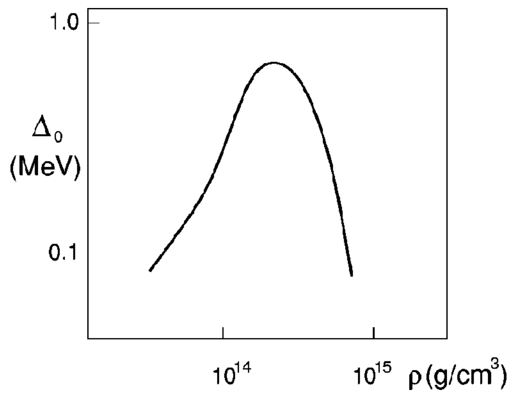

FIG. 1. The energy gap as a function of matter density inside a neutron star [6]. Superconductivity occurs in some range of densities, $10^{14} < \rho < 6\times 10^{14}$ g/cm^3.

expands; then the proton condensate may oscillate around the minimum. If the magnetic field is close to critical, $B \sim B_c$, the two minima in the potential shown in Fig. 2 are nearly degenerate and no subsequent coherent motion of the condensate ϕ takes place after the transition through bubble nucleation. However, for a smaller magnetic field, the so-called ''escape point'' ϕ_e is different from the global minimum ϕ_0 and the scalar condensate may oscillate around the minimum.

During oscillations of the order parameter ϕ, a photon has effectively a time-dependent mass proportional to the value of ϕ. This may, in some cases, signal a copious production of gamma quanta through parametric resonance [9]. In this paper we study the parametric resonance both analytically and numerically for some sample values of the nuclear density and magnetic field in the range from 10^{12} to 10^{15} G.

III. POTENTIAL AND EQUATION OF MOTION

A natural framework for describing the relaxation of the proton condensate after the phase transition is time-dependent Ginzburg-Landau theory [10,11]. The applicability of such a description is limited to cases where the Joule heat loses are small [12]. Otherwise, the interactions become essentially non-local and a simple equation of motion for a scalar field ceases to be valid.

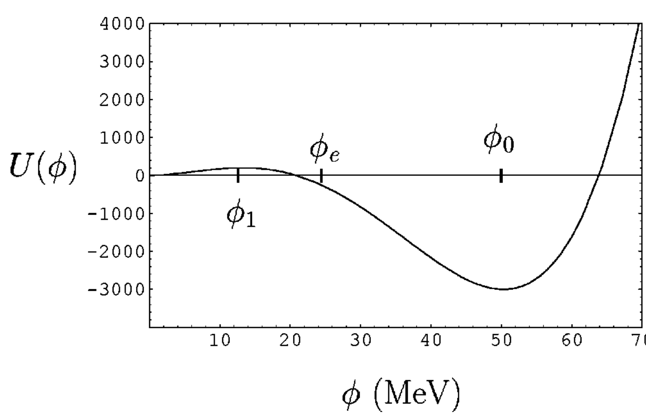

FIG. 2. Effective potential for the proton condensate, in units of MeV^4. The small bump at ϕ_1 is due to a magnetic field B.

Here we will assume the validity of the time-dependent Ginzburg-Landau theory [11,12] and will apply it to the superconducting proton condensate inside a neutron star. One should take this approach with a grain of salt given the lack of empirical knowledge with respect to proton superconductivity in nuclear matter. However, we hope that—as crude an approximation as this may be—it may help identify the main features of the resonant production of gamma rays.

We write the equation of motion for the order parameter as

$$\ddot{\phi} + \frac{8\varepsilon_F}{3c}\dot{\phi} - \frac{2\varepsilon_F}{3cm_*}\nabla^2\phi + U'(\phi) = 0, \tag{1}$$

where $\varepsilon_F = p_F^2/2m_*$ is the Fermi energy of the proton condensate and $c = [28\zeta(3)/3\pi^3]\varepsilon_F/T_c$ is a constant characterizing the condensate. The friction term then has a magnitude $8\varepsilon/3c \simeq 7.37T_c \simeq 4.2\Delta_0$, of the order of a few MeV, which is comparable to the oscillating frequency ω of the condensate around its minimum ϕ_0, and therefore cannot be ignored. On the other hand, the gradient term in Eq. (1) is negligible in our case [4], and thus we are dealing with a (locally) homogeneous condensate.

The effective potential for the order parameter ϕ can be determined from the physical properties of the condensate. After the acoustic wave has restored the symmetry, the energy density subsides and a new superconducting phase transition takes place. The difference in energy density associated with the proton condensate is [13]

$$\Delta U(\phi_0) = \frac{m_* p_F}{4\pi^2}\Delta_0^2, \tag{2}$$

where Δ_0 is the proton energy gap, and m_* is the effective proton mass in nuclear matter, which is somewhat lower than the bare proton mass m_p. The equilibrium value of the order parameter is given by $\phi_0^2 = n_p/2m_*$, where $n_p = Y_p\rho/m_p$ is the number density of protons inside the neutron star, which make up a fraction $Y_p \approx 0.03$ of all baryons.

The way the superconducting phase transition occurs depends very strongly on the presence of a magnetic field B. Such a field creates a barrier in the effective potential between the symmetric and the superconducting phases. The height of the barrier is approximately given by the total energy density in the magnetic field,

$$\Delta U(\phi_1) = \frac{B^2}{4\pi} \simeq 200\left(\frac{B}{10^{15}\ \mathrm{G}}\right)^2\ \mathrm{MeV}^4. \tag{3}$$

This barrier makes the phase transition first order, with the creation of bubbles of the superconducting phase. After bubble nucleation, the proton condensate oscillates and soon settles (due to friction) at its equilibrium value ϕ_0. We have shown in Fig. 2 the effective potential $U(\phi)$ for a region inside the neutron star with matter density $\rho = 10^{14}$ g/cm^3 and magnetic field $B = 10^{15}$ G, which corresponds to $\Delta_0 \simeq 0.35$ MeV, $\phi_0 \simeq 50.3$ MeV, $\Delta U(\phi_0) \simeq 3000$ MeV^4 and $\Delta U(\phi_1) \simeq 200$ MeV^4.

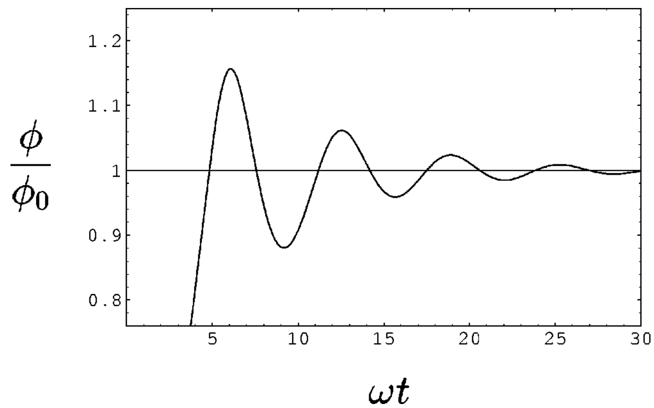

FIG. 3. The oscillations of the order parameter associated with the proton condensate around the minimum of the effective potential. Time is given in units of the effective frequency. Because of the friction term, the amplitude of the proton condensate diminishes in a few oscillations.

Depending on the relative size of the friction term, the condensate settles at ϕ_0 after a few oscillations, or without oscillations at all. We show in Fig. 3 the oscillations of the order parameter ϕ in the case of the parameters of Fig. 2, which give a friction coefficient of the order of $4.2\Delta_0 \simeq 0.31\omega$ and $\omega \simeq 4.7$ MeV. This allows a few oscillations to take place, which is crucial for the resonant production of photons, as we will see in the next section.

IV. RESONANT PHOTOPRODUCTION

The proton condensate comprises Cooper pairs of charge $2e$, which couple to the electromagnetic field through the term $(2e)^2\phi^2 A_\mu A^\mu$ in the unitary gauge, in which the scalar field ϕ is real. As the condensate oscillates around ϕ_0, it induces an effective periodic mass, which in some cases may stimulate parametric resonant production of photons [9]. The mechanism is similar to that discussed in connection with axion clumps [14], as well as *preheating* after inflation [15], where explosive production of bosons may occur under special circumstances [16].

We will write the gauge field as $A_\mu(x) = \chi(x) e_\mu(x)$, where $e_\mu(x)$ is a polarization vector and $\chi(x)$ can be expanded in Fourier modes, $\chi_k(t)$, which satisfy the evolution equation

$$\ddot{\chi}_k + [k^2 + 2(2e)^2\phi^2(t)]\chi_k(t) = 0, \tag{4}$$

with an effective mass proportional to the condensate

$$\phi(t) \simeq \phi_0[1 + \Phi\ \exp(-\epsilon\omega t/2)\sin\omega t], \tag{5}$$

where $\Phi = \phi_e/\phi_0$ and $\epsilon \simeq 4.2\Delta_0/\omega$ is the decay constant for the condensate oscillations. Equation (4) can be written as a Mathieu equation [17] with coefficients $(z = \omega t/2)$

$$A_k = \frac{4k^2}{\omega^2} + 4q_0, \tag{6}$$

$$q(z) = 4q_0\Phi\ \exp(-\epsilon z), \tag{7}$$

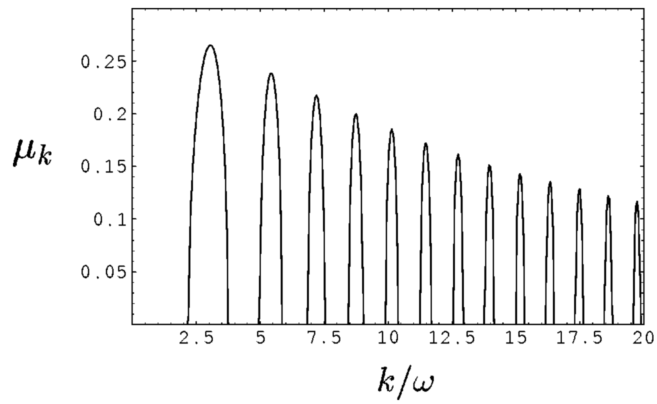

FIG. 4. The growth parameters μ_k as a function of momenta in units of the effective frequency, k/ω, after one oscillation of the condensate.

$$q_0 = 8e^2\frac{\phi_0^2}{\omega^2} = 32\pi^2\alpha_{\rm em}\frac{\phi_0^2}{\omega^2}. \tag{8}$$

Note that the effective parameter $q(z)$ that determines the strength of the resonance decreases exponentially with time. If ϵ is too large, the parametric resonance is weak, and this mechanism is inefficient. On the other hand, if ϵ is small, then the condensate oscillates several times and causes an explosive production of gamma rays with energy of a few MeV.

At density $\rho \approx 10^{14}$ g/cm^3 and magnetic field $B \sim 10^{15}$ G, parametric amplification of the MeV photons can take place. In this case, $\epsilon = 0.31$ and $4q_0 \simeq 340$. We have plotted μ_k as a function of k, after one oscillation of the condensate, in Fig. 4, from which we can deduce the spectrum

$$n_k \simeq \frac{1}{2}\exp(\mu_k \omega t). \tag{9}$$

Nuclear matter is opaque for photons with energy of the order of the electron chemical potential, which limits the spectral band in which the modes are amplified. The spectrum n_k has, therefore, an ultraviolet cutoff at the momentum associated with the electron Fermi energy inside the neutron star, beyond which photons are absorbed and the resonance shuts down. This occurs at wavelengths $k_c \simeq 100 m_e = 51$ MeV. For the model discussed above, with frequency of oscillations $\omega = 4.7$ MeV, this corresponds to $k/\omega \simeq 11$.

Furthermore, after a few oscillations, a back reaction occurs; that is, the number of created photons is so large that it dominates the frequency of oscillations of the proton condensate, $m^2 = \omega^2 + 8e^2\langle\chi^2\rangle$. The back reaction sets in when [15]

$$\langle\langle\chi^2\rangle\rangle = \frac{1}{2\pi^2}\int_0^{k_c} dk\, k^2 \frac{n_k(t)}{\omega_k} \simeq \frac{\omega^2}{8e^2} \sim \omega^2, \tag{10}$$

where $\omega_k^2 = k^2 + 8e^2\phi^2(t)$ is the effective frequency of the mode k, and the integration is up to the physical cutoff, k_c. For the model in hand, we find that the back reaction takes

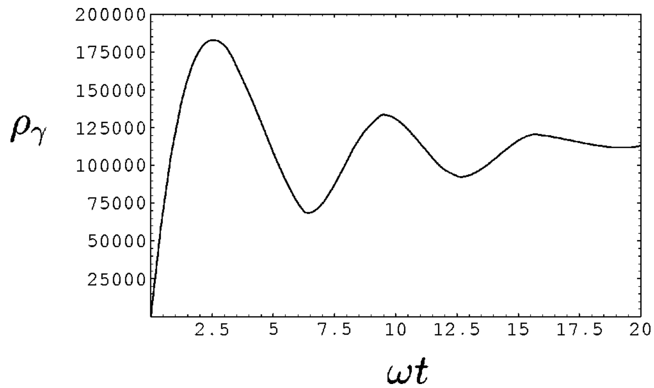

FIG. 5. The energy density, in units of MeV^4, of gamma ray photons produced via parametric resonance, as a function of time.

place after about one oscillation, much before the friction term in Eq. (1) has significantly decreased the oscillation amplitude of the condensate.

The total energy density in photons produced during the resonance is, therefore,

$$\rho_\gamma(t) = \frac{1}{2\pi^2}\int_0^{k_c} dk\, k^2\, \omega_k\, n_k(t). \tag{11}$$

We have plotted this energy density in Fig. 5. Here $\rho_\gamma(t)$ reaches an asymptotic value after a few oscillations. Because of the back reaction, the energy density produced is that after the first oscillation, of the order of $10^5\ \mathrm{MeV}^4$. It may seem like a large value, but actually the energy density produced via this mechanism is just a small fraction of the neutron star density, $\rho_\gamma \simeq 8\times 10^4\ \mathrm{MeV}^4 \simeq 10^{-4}\rho$.

We have analyzed the resonant photoproduction numerically for magnetic field from 10^{12} G to 10^{16} G and found no significant deviation from the sample values described above. The main effect of the magnetic field is to produce a potential barrier at $\phi = \phi_1$, as shown in Fig. 2. A large magnetic field may, however, have some other effects on nuclear matter. For example, it can modify the particle composition, in particular the distribution of protons and electrons [18]. This, in turn, can affect the resonant photoproduction. Other parameters, such as the electron chemical potential, the proton fraction Y_p, etc., can differ significantly from the sample values we took. However, our numerical analyses show that a resonant production of gamma rays from the oscillating charged condensate is possible for a wide variety of parameters.

V. IMPLICATIONS FOR THE OBSERVATION OF GAMMA-RAY BURSTS

The MeV-energy gamma rays produced in each cycle of oscillations cannot decay the way they would decay in vacuum, through electron-positron pair production, because the electrons are highly degenerate inside the neutron star. The Pauli exclusion principle prevents a production of electrons with energies less than the electron chemical potential, which is of the order of 100 MeV. Compton scattering off the electrons and protons near the Fermi surface is kinematicly suppressed but is not forbidden. The corresponding mean free path is $\lambda \sim 10^{-9}$ cm. Comptonization generally preserves the number of photons and tends to equalize the temperatures of electrons and photons [19]. Given enough time, the photons would diffuse out of the star. However, since the merger of the two neutron stars is characterized by very short time scales, all the gamma rays non-thermally produced by the coherent oscillations of the proton condensate remain inside the star when the fireball erupts. At that point they can leak out and be observed as a short gamma-ray burst or, more likely, as a component of the GRB.

The energy stored in the non-thermal bath of gamma ray photons inside the neutron star is $E_\gamma = \rho_\gamma V \simeq 3\times 10^{50}$ erg. If the collision of neutron stars releases these photons within a time scale of the order of $1\ \mathrm{ms} \lesssim \tau \lesssim 0.1$ s, then the power generated is of the order of $10^{52\pm 1}$ erg/s, which corresponds to the power emitted in the observed gamma ray bursts [1].

Resonant photoproduction is active in a spherical shell with density 10^{14}–10^{15} g/cm^3, which contains most of the neutron star mass [19] and where the acoustically driven superconducting phase transitions can take place.

VI. CONCLUSION

Strong mechanical shock waves, such as those expected to be generated by a collision of two neutron stars, can cause repetitious superconducting phase transitions in nuclear matter. The relaxation of the proton condensate to its potential minimum can result in the non-thermal resonant production of γ quanta in the MeV energy range.

There are some important consequences. The presence of energetic gamma quanta in nuclear matter, relatively transparent to photons thanks to electron degeneracy, can affect the equation of state. This, in turn, can affect the dynamics of the neutron star coalescence.

In addition, the gamma rays trapped until the onset of the fireball, but then released by the explosion, can also contribute to gamma-ray bursts.

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